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LETTER

Iterative Decoding with LDPC Based Unitary Matrix Modulated OFDM with Splitting the Diagonal Components over the Coherence Bandwidth

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SUMMARY In this letter, we propose an iterative decoding with LDPC based unitary matrix modulated OFDM with splitting the diagonal components over the coherence bandwidth. The proposed system can obtain a frequency diversity gain by splitting the diagonal components of unitary matrix modulated symbols, and also obtain large coding gain by using LDPC code.

key words: *LDPC, OFDM, coherence bandwidth, unitary matrix*

1. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) systems have recently attracted considerable attention as a fourth generation mobile communication system due to the parallel signal transmission using many subcarriers that are mutually orthogonal. Moreover, since the frequency spacing of each subcarrier is minimum, OFDM can treat a frequency-selective fading as a flat fading for each subcarrier. One of the approaches to improve OFDM performance is the use of the spatio-temporal processing such as space-time block code (STBC) at the transmitter or receiver with multiple antennas. However, STBC is required the linear operation to separate and decode simultaneously received signals from the different transmit antennas. Recently unitary space-time modulation (USTM) and USTM/OFDM have been proposed to perform space-time diversity without linear operation [1], [2]. Since components except diagonal component in unitary matrices 0, received signals transmitted from different transmit antenna, are received simultaneously and the received signal can be separated without linear operation. However, if the fading is fast or large number of antenna elements are used, estimating the fading coefficients between each pair of transmitter and receiver antenna elements become difficult and inefficient. Moreover, synchronization is also critical problem in multiple antenna systems. When we consider M transmit antennas and N receive antennas, the diversity order is (M, N), and then, the channel estimation processing and the synchronization are required $M \times N$ times processing. On the other hand, in a single antenna system, the channel estimation processing and the synchronization are required only one time processing. However, the performance of a single antenna system is limited due to the channel capacity theory. If it is possible to increase the system performance with a single antenna, this system is a practical method of attaining diversity. To reduce previous problems and increase the performance of a single antenna system, we propose unitary matrix modulated OFDM with splitting the diagonal components over the coherence bandwidth in a single antenna. In USTM/OFDM, these unitary matrices may be viewed as a multiple antenna extension. USTM signal is a matrix, whose rows are transmitted from M antenna elements and mutually orthogonal to each other. In the transmitter, the data stream is divided into bit sequences that consist of $R \cdot M$ bits, where R and M denote information bits per parallel symbol to be transmitted, and the number of transmit antennas, respectively. Each $R \cdot M$ bit sequence is mapped into the constellation $\mathbf{U}(l)$ $(0 \leq l \leq L-1)$ selected from $L = 2^{RM}$. The constellation of USTM can be written as $\mathrm{diag}(\ \mathbf{U}(l)\ ) = \{e^{i\frac{2\pi l}{L}}, \cdots, e^{i\frac{2\pi l}{L}}\}$, where $\mathbf{U}(l)$ is $M \times M$ unitary matrix with only diagonal component non-zero and "diag" is the diagonal operator [2]–[4]. However, the performance of USTM largely depends on the constellation of USTM matrix. It requires the design of relatively large constellations of matrix-valued signals according to a criterion that differs markedly from the traditional maximum-Euclidean-distance criterion. Recently, novel unitary matrices have been proposed [5], [6]. Note that we do not claim the optimality of the USTM, but rather, we argue that the USTM with its flexible scalability, and high performance. Therefore, in this paper, we consider the conventional unitary matrix with only diagonal component non-zero for simulation. With considering only diagonal components of unitary matrix and splitting the frequency domain in a single antenna, this system can be improved like the conventional USTM. This is because the split diagonal components also obtain different channel responses that called the frequency selective order η due to the frequency selective fading. From the information theoretical perspective, error control code with a bit interleaving can be further increased the diversity order over the η [7]. However, the effective length of unitary matrix is more shorter than the interleaving span and the frequency selective order. Therefore, the received data stream is included apparent the channel variation. It means that it is possible to obtain a frequency diversity using forward error correction (FEC) with interleaving. Since without considering the channel state information (CSI), FEC with block interleaving is not optimal

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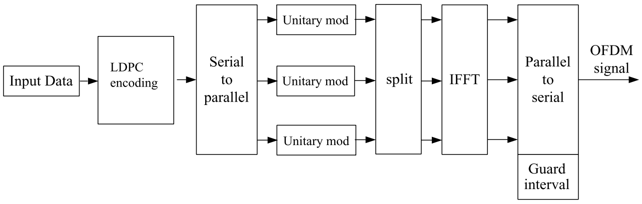
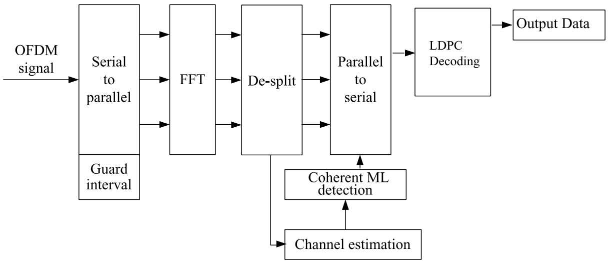


(a) Transmitter



(b) Receiver

Fig. 1 Proposed LDPC coded UMM-S/OFDM system.

[8], [9]. Recently, Low density parity check (LDPC) code is strongly attended and studied due to simple decoding property [10]. LDPC code is a linear code, and is composed of huge size and very sparse parity check matrix. LDPC code can achieve the BER performance which is very close to the Shannon limit as well as turbo code when the LDPC code has enough long code length. In this letter, we consider a LDPC code as a FEC for UMM-S/OFDM and investigate the performance improvement of the proposed system. This letter is organized as follows. The proposed system is described in Sect. 2, In Sect. 3, we show the simulation results. Finally, the conclusion is given in Sect. 4.

2. Proposed System

In a single transmit and receive antenna system, the channel response of frequency domain at the k-th sub-carrier can be expressed as

$$H(k) = \sum_{p=0}^{P-1} h_p(k) e^{j2\pi kp/K} = \mathcal{H}(k)^H \alpha(k) \tag{1}$$

where $\mathcal{H} = [h_0, h_1, ..., h_{P-1}]^H$ is P-sized vector containing the time responses, P is the number of channel paths, and $\alpha(k)$ is FFT coefficient. The received signal $Y(k)$ of the k-th sub-carrier at receiver side is given by

$$Y(k) = H(k)X(k) + N(k) \quad k = 1, \cdots\cdot, K \tag{2}$$

where $X(k)$ is the diagonal component of unitary matrix with splitting over the coherence bandwidth, and N is white Gaussian noise. The channel response at a particular sub-carrier frequency is not supposed to be totally different from its neighboring frequencies, and hence, they must have correlation which depends on the coherence bandwidth of the channel B_c. However, the channel response between sub-carriers split over the coherence bandwidth shows totally different. In this case, we can expect the frequency diversity with splitting the coherence bandwidth. Therefore, the diagonal components of unitary matrix $X(k)$ with splitting over the coherence bandwidth can obtain diversity gain in a single antenna system. After de-splitting the received signal and channel estimation, the frequency domain signal Y is divided into M bits. Here, we consider the same structure of unitary matrix for $M \times M$ like USTM [2]. Each M bits of frequency domain signal is demodulated by ML estimator. The ML decision rule of the signal model (2) is given by

$$\hat{U} = arg\ \min \sum_{k=1}^{K} \Bigg| Y(k) - H(k) \cdot \sum_{l=1}^{L} \mathrm{diag}(U)_{mod(k,M)+1}(l) \Bigg|^2 \tag{3}$$

where $\mathrm{diag}(U)$ is the diagonal of unitary matrix U, $\sum_{l=1}^{L} \mathrm{diag}(U)_{mod(k,M)+1}(l)$ is the diagonal component of unitary matrix and L is the number of unitary matrices. From Eq. (3), the neighboring signals of $\sum_{l=1}^{L} \mathrm{diag}(U)_{mod(k,M)+1}(l)$ without splitting must have correlated channel responses. However, the split signals over the coherence bandwidth obtain totally different channel responses. It means that UMM-S/OFDM in a single antenna system obtains a frequency diversity gain. Moreover, unitary matrix has short effective length. Therefore, it is possible to increase the performance using FEC with interleaving. Here, we consider LDPC code as a FEC and block interleaving. LDPC code is a linear code, and is composed of huge size and very sparse parity check matrix. LDPC code can achieve the BER performance which is very close to the Shannon limit as well as turbo code when the LDPC code has enough long code length. From these reasons, LDPC coded system is attractive. However, the performance of LDPC codes increases with code length. For large code lengths, the standard representation of matrices requires too much memory. For 10000 × 5000 LDPC codes, we may assume that the density of 1's $\mathcal{G}$ is 0.5. There are 25×10^6 1's in $\mathcal{G}$. In this case, 25×10^6 addition (XOR) operations are required to encode one codeword. It means that the encoder requires too much memory. Since the minimum distance of binary LDPC codes increases linearly with the code length, further performance improvement is possible by increasing the code length. Note that we do not claim the optimality of the proposed LDPC, but rather, we argue that the proposed LDPC with its low decoding complexity and high performance like [11], [12]. In this letter, we consider a LDPC code as a FEC like previous studies with small code length such as $\mathcal{G} = \{128, 64\}$. To perform LDPC decoding, the sum-product algorithm is an iterative decoding that transfers probability information in row and column direction. The sum-product algorithm is widely considered as a decoding algorithm [13]. The probability information is used to decide whether a bit is 0 or 1. $r_{m,n}$ and $q_{m,n}$ are probability information transferred from row direction using (4) and column direction using (5), respectively.

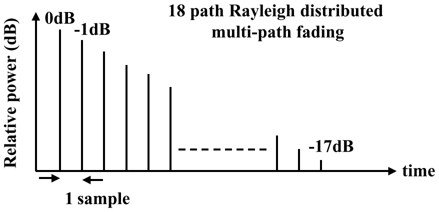


Fig. 2 Channel model.

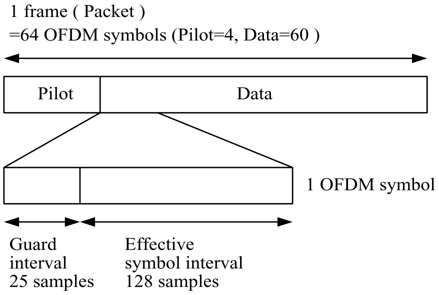


Fig. 3 Packet structure.

$$r_{m,n}(a) = \mathcal{K} \sum_{i \in A(m)\backslash n} \prod_{\tilde{n} \in \sum c_i = aA(m)\backslash n} q_{m,\tilde{n}}(\tilde{c}_{\tilde{n}}) P(y_{\tilde{n}}|c_{\tilde{n}}) \tag{4}$$

$$q_{m,n}(\tilde{a}) = \tilde{\mathcal{K}} \prod_{\tilde{m} \in B(n)\backslash m} r_{\tilde{m},n}(\tilde{a}) \tag{5}$$

where a and $\tilde{a}$ are 0 or 1. Constant $\mathcal{K}$ and $\tilde{\mathcal{K}}$ are defined so that $r_{m,n}(0) + r_{m,n}(1) = 1$, and $q_{m,n}(0) + q_{m,n}(1) = 1$ are met, respectively. $A(m)\backslash n$ is a set except variable n of column indexes the value of which is one in m-th row of parity check matrix $\mathcal{G}$. Conversely, $B(n)\backslash m$ is a set except variable m of row indexes the value of which is one in n-th column of $\mathcal{G}$. After posterior probability of each bit is calculated from $r_{m,n}$ using Bayes' theorem, the value of each bit is estimated.

3. Computer Simulated Results

Figure 1 shows UMM-S/OFDM with LDPC code for N_c = 128 subcarriers in a single antenna. In the transmitter, data stream is first encoded. The coded bits are then S/P transformed, and UMM-S/OFDM system maps the diagonal components of unitary matrix with splitting over the coherence bandwidth. In this case, the diagonal components of unitary matrix obtain the different channel responses, so the UMM-S/OFDM system can be obtained diversity gain in a single antenna. The performance of LDPC based UMM-S/OFDM is simulated in Rayleigh fading channel. In the channel model as shown in Fig. 2, L = 18 path Rayleigh fadings have exponential shapes with path separation $T_{path} = 150nsec$. This case causes a severe frequency selective fading channel. Table 1 shows the simulation parameters. Figure 3 shows packet structure. Packet consists of 128 sub-carriers and 64 OFDM symbols (number of pilot signals: N_p = 4, number of data: N_d = 60). LDPC codes (rate R = 1/2, $\mathcal{G}$ = {128, 64}) are used. Figure 4 shows BER performance of the conventional OFDM without diversity, UMM-S/OFDM with splitting sizes as 8, 16, and 64 in a single antenna, USTM/OFDM with 2Tx and 1Rx at Doppler frequency of 10 Hz. Without splitting, UMM-S/OFDM system shows the same BER performance like the conventional OFDM. On the other hand, the BER performance of UMM-S/OFDM system shows better BER than that of the conventional OFDM. Particularly, when increasing the splitting sizes, the diagonal components of unitary matrix obtain different channel impulse response. Thus, UMM-S/OFDM system can achieve a frequency diversity. Figure 5 shows BER performance of uncoded OFDM and UMM-S/OFDM with 64 samples splitting, and LDPC coded OFDM and UMM-S/OFDM with 64 samples splitting in a single antenna at Doppler frequency of 10 Hz. UMM-S/OFDM can increase the BER performance significantly with splitting of the diagonal components of unitary matrix without considering FEC. When we consider LDPC code for OFDM (LDPCC-OFDM) with block interleaving and UMM-S/OFDM (LDPCC-UMM-S/OFDM) with 64 samples splitting using block interleaving, LDPC coded UMM-S/OFDM system shows 6dB gain compared with LDPC

Table 1 Simulation parameters.

Data Modulation	BPSK
Demodulation	Coherent ML detection
Data rate	15.68 Msymbol/s
Frame size	64 symbols (N_p = 4, N_d = 60)
Number of carriers	128
Guard interval	25 sample times
split size	8,16,64 samples
Fading	18 path Rayleigh fading
Doppler frequency	10 Hz
FEC	LDPC code (R=1/2, $\mathcal{G}$={128,64})
Interleaving	Block interleaving (1 × 128)
Antenna	Tx=1, Rx=1

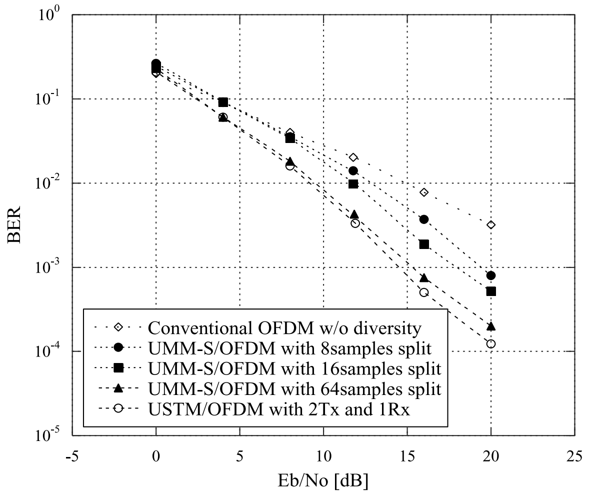


Fig. 4 BER performance of the conventional OFDM without diversity, UMM-S/OFDM with splitting sizes as 8, 16, and 64 in a single antenna, USTM/OFDM with 2Tx and 1Rx at Doppler frequency of 10 Hz.

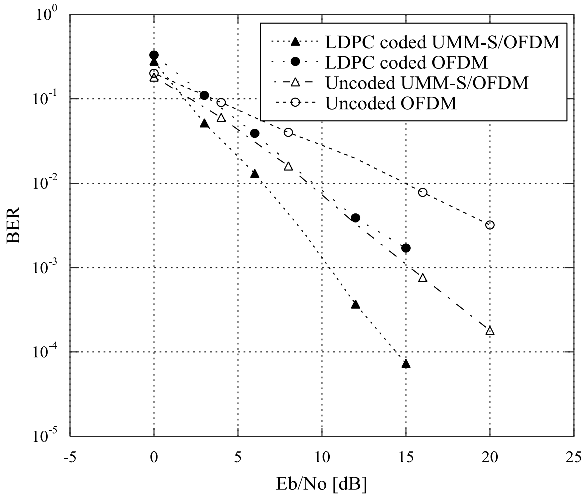


Fig. 5 BER performance of uncoded OFDM and UMM-S/OFDM with 64 samples splitting, and LDPC coded OFDM (LDPCC-OFDM) and UMM-S/OFDM (LDPCC-UMM-S/OFDM) with 64 samples splitting in a single antenna at Doppler frequency of 10 Hz.

coded OFDM. This is because the splitting is basically same effect like interleaving, but the effective length of unitary matrix is short. Therefore, the received data streams are included apparent the channel variation due to short effective length of unitary matrix. Without considering the CSI, error correction with block interleaving is not optimal. From these reasons, LDPCC-UMM-S/OFDM with block interleaving system shows better BER than that of LDPCC-OFDM with block interleaving.

4. Conclusion

In this letter, we have proposed the LDPC coded OFDM using unitary matrix modulation with splitting over the coherence bandwidth (LDPCC-UMM-S/OFDM) in a single antenna, and investigated the performance improvement of the proposed system. From the simulation results, it is shown that the proposed system achieves better BER performance than that of the conventional system.

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